OTTO THEORY MANUAL

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I. INTRODUCTION

OTTO is a program which simulates the ocean tow of a jacket on a barge. Its primary objective is to provide the engineer with an analytical tool to evaluate the stresses in both the jacket and in the barge which result from the various seastates that the tow might encounter.

The purpose of this document is to describe the theories and assumptions made by the program. It is not the purpose of this paper to fully develop all of the theory; for example, the elastic modelling of the jacket/barge structure is accomplished via standard finite element techniques which are widely known, and which will thus be only briefly outlined. In areas where the theory is not widely known or universally accepted (e.g., joint type classification), more detail will be given.

I.A **ANALYSIS OVERVIEW**

The basic function of OTTO is to perform a stress analysis of a barge/jacket combination subject to a seastate. Thus, the governing equation of motion is

$$\underline{M}\,\ddot{\underset{\sim}{\Delta}} + \underset{\sim}{K}\,\underset{\sim}{\Delta} = \underset{\sim}{\hat{g}} + \underset{\sim}{\hat{d}} + \underset{\sim}{\hat{k}} \tag{I-1}$$

where

A = structural deflections

$\underset{\approx}{K}$, $\underset{\approx}{M}$ = structural stiffness and mass matrices

$\hat{g}$ = sea/structure interaction forces

$\hat{d}$ = dead loads

$\hat{k}$ = other applied loads (e.g., wind, current, etc.)

Next, suppose that we can decompose Δ into two parts

$$\underset{\sim}{\Delta} = \underset{\sim}{x} + \underset{\sim}{u} \tag{I-2}$$

where x is the rigid body motion of the system and u is the elastic deformation. In OTTO we further suppose that

1.) The acceleration of the elastic deformation can be neglected,

2.) The sea/structure forces, $\hat{g}$, are independent of the deformation, u, and

3.) The jacket is not in the water, so that the sea forces act only on the barge.

Then, writing $\underset{\sim}{x}$ in terms of the six rigid body degrees of freedom (DOF), δ_i, i=1,6, we get

$$\underset{\sim}{x} = \underset{\approx}{\bar{x}}\,\underset{\sim}{\delta} \tag{I-3}$$

where each of the six columns of $\underset{\approx}{x}$ represent the structural displacement, Δ, due to a unit rigid body motion of the system. Combining (I-l) through (I-3) yields

$$\underset{\approx}{M}\,\underset{\approx}{\bar{x}}\,\underset{\sim}{\ddot{\delta}} + \underset{\sim}{k}\,\underset{\sim}{u} = \underset{\sim}{\hat{g}} + \underset{\sim}{\hat{d}} + \underset{\sim}{\hat{k}} \tag{1-4}$$

which can be decomposed into two separate problems: one of rigid body dynamics and one of a static stress analysis. Thus, we obtain

$$\underset{\approx}{\bar{M}}\,\underset{\sim}{\ddot{\delta}} = \underset{\sim}{\bar{g}} + \underset{\sim}{\bar{d}} \tag{1-5}$$

$$\underset{\approx}{K}\,\underset{\sim}{u} = \hat{g} + \underset{\sim}{\hat{d}} + \underset{\sim}{\hat{k}} - \underset{\approx}{M}\,\underset{\approx}{\bar{x}}\,\underset{\sim}{\ddot{\delta}} \tag{1-6}$$

where

$$\begin{aligned}
\underset{\approx}{\bar{M}} &= \underset{\approx}{\bar{x}}^{T}\,\underset{\approx}{M}\,\underset{\sim}{x} \\
\underset{\sim}{\bar{g}} &= \underset{\approx}{\bar{x}}^{T}\,\underset{\approx}{\hat{g}} \\
\underset{\sim}{\bar{d}} &= \underset{\approx}{\bar{x}}^{T}\,\underset{\sim}{\hat{d}}
\end{aligned} \tag{1-7}$$

The first equation is a six by six system to be solved for rigid body response, $\underset{\sim}{\ddot{\delta}}$, and the second equation is solved for the elastic deformation, $\underset{\sim}{u}$.

II. VESSEL RIGID BODY MOTIONS

As was outlined in Section I, the stress analysis is based upon loads derived by first considering the response of the barge/jacket combination as a rigid body. In this section, we will briefly outline the method of determining the forces which the sea imposes on the structure. Since we have assumed that the jacket is not in the water, the problem is then that of a floating vessel.

II.A SEA/STRUCTURE INTERACTION

The analysis of the interaction of a floating body with the surrounding fluid has a lengthy history. In particular, Salvesen, Tuck, and Faltinsen [1] present not only a concise statement of the current state of the art, but also a reasonable history of the subject. Most of the following is based on this work. For purposes of calculating the sea/structure interaction, we will assume that the barge can be considered to be composed of a collection of rigid elements, each of which is prismatic. We will also assume that the flow induced by the motion of the vessel (element) parallel to the longitudinal axis is negligible. Mathematically, this can be shown to be the case provided the wave lengths of generated waves are short in comparison to the length of the barge. The assumption of no induced parallel flow allows us to reduce the mathematical problem from a three dimensional one to a sequence of two dimensional ones. In addition, we will suppose

1.) The motions of each element are small,

2.) The fluid is incompressible and invisid, and

3.) The flow is irrotational.

These assumptions are standard in the study of ship motions, and they allow us to reduce the problem to one of linear potential theory.

In particular, for each element of the structure, the force due to interaction is obtained by integrating the pressure, p, over the submerged portion of the element. Here, the pressure is given by the linearized Bernoulli equation

$$P = -\rho \left(\frac{\partial \phi}{\partial t} + g_c z \right) \qquad (II-1)$$

where ρ is the mass density of the fluid, g_c is the acceleration of gravity, and ϕ is the velocity potential for the flow which must satisfy

$$\nabla^2\phi = 0, \text{ on the exterior of the element,} \tag{II-2}$$

$$\frac{\partial^2\phi}{\partial t^2} + g_c \frac{\partial\phi}{\partial z} = 0, \text{ on the free surface,} \tag{II-3}$$

$$\nabla\phi \cdot \underset{\sim}{n} = \underset{\sim}{v}, \text{ on the submerged surface of the section,} \tag{II-4}$$

and an appropriate radiation condition. These conditions simply state that the velocity potential must satisfy LaPlace's equation, the linearized free surface condition, the velocity of the flow at each point on the element must equal the velocity of the section at that point, and the generated flow must behave as an outgoing wave at infinity.

As is traditionally the case, we will consider the prob em in the frequency domain. In other words, suppose that the velocity potent al has the form

$$\phi = \text{Re}\ (\phi e^{i\omega t}) \tag{II-5}$$

where $i = \sqrt{-1}$, and ϕ is a complex potential which is independent of time. At this point, it is convenient to decompose the prob em into seven pieces. These pieces correspond to a wave induced by a motion of each degree of freedom, and by the scattering of the incident wave by the vessel. Thus,

$$\phi = \phi_0 \sum_{j=1}^{7} \phi_j \delta_j \tag{II-6}$$ [1]

where ϕ_0 is the incident wave potential

$$\phi_0 = \frac{g_c \eta}{} \exp[-k(z + i x \cos\beta + i y \sin\beta] ,$$

$$k = \frac{\omega^2}{g_c} , \tag{II-7}$$

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[1] For clarity, in this section, we will use ind cial notation when referring to components of tensors.

β = the angle that the incident wave makes with the X-axis,

η = the incident wave height,

δ_j = the motion of each degree of freedom, (II-7 cont.)

$\delta_7 = \eta$,

and each potential ϕ_j satisfies

$$\underset{\sim}{\nabla}\phi_j \cdot \underset{\sim}{n} = i\omega n_j \qquad \text{(II-8)}$$

where $\underset{\sim}{n}$ is the unit outward normal to the surface, and n_j is the generalized normal given by

$$(n_1, n_2, n_3) = \underset{\sim}{n}$$

$$(n_4, n_5, n_6) = \underset{\sim}{r} \times \underset{\sim}{n} \text{ , and} \qquad \text{(II-9)}$$

$$n_7 = \underset{\sim}{\nabla}\phi_0 \cdot \underset{\sim}{n}$$

r being the position vector.

If the preceding results are combined with (II-l), and the results integrated over the vessel surface, we find that the generalized force on the element can be expressed as

$$g_i = -\rho \int_S [i\omega\ (\eta\phi_0 + \sum_{j=1}^{7} \phi_j\delta_j\ ) + g_c z]\ n_i\ dA\ , \qquad \text{(II-10)}$$

or, if this is combined with (II-8),

$$g_i = -\rho \int_S zn_i\ dA - \eta\rho i\omega \int_S (\phi_0 + \phi_7)\ n_i\ dA$$
$$+ \rho\omega^2 \sum_{j=1}^{6} [\int_S \underset{\sim}{\nabla}\phi_j \cdot \underset{\sim}{n}\ \phi_i\ dA]\ \phi_j \ . \qquad \text{(II-11)}$$

Notice that the first appears to be constant. This is not, however, the case. Instead, it can be shown that, for small motions,

$$-\rho g_c \int_S z n_i \, dA = s_i - R_{ij}\delta_j \tag{II-12}$$

where s_i is the generalized force on the vessel due to buoyancy with the vessel in its mean position, and R_{ij} is a matrix of hydrostatic restoring coefficients. The second term does not depend on the motion, but is linear in the wave amplitude. This is the force on the vessel due to the presence of waves, and we will denote it as

$$q_i = -i\omega\rho \int_S (\phi_0 + \phi_7) \, n_i \, dA \; . \tag{II-13}$$

Finally, since ϕ_i is complex, we will define

$$H_{ji} = \mathrm{Re}\left(\rho \int_S \nabla\phi_j \cdot \underset{\sim}{n} \, n_i \, dA\right), \text{ and}$$

$$D_{ji} = \mathrm{Im}\left(\frac{\rho}{i\omega} \int_S \underset{\sim}{\nabla}\phi_j \cdot \underset{\sim}{n} \, n_i \, dA\right). \tag{II-14}$$

Combining (II-12) through (II-14), we find that the total force on the vessel can be represented as

$$g_i = s_i + \eta q_i - \sum_{j=1}^{6} [-\omega^2 H_{ji} + i\omega D_{ji} + R_{ji}] \, \delta_j \tag{II-15}$$

which is the desired result. Unfortunately, the problem remains to solve for the seven potentials, ϕ_j, so that the quantities in (II-15) can be evaluated.

While there are several techniques available which will solve for the seven unknown potentials, the one which appears to be the most satisfactory is the one developed by Frank [2]. His technique is based on the results of John [3]. In particular, except for the set of discrete frequencies, the solution of each of our problems can be expressed as

$$\phi_j = \int_s Q_j \ G \ ds \tag{II-16}$$

where Q_j is the distribution of source intensities, and G is the potential of a pulsating source along the surface. Since this representation satisfies all the conditions of our problem except (II-B), the final solution is obtained by solving the integral equation

$$[\nabla \cdot \int_s Q_j \ G \ ds] \cdot n = i\omega n_j \ . \tag{II-17}$$

To solve this equation, Frank assumes that the source intensities can be approximated as constants over segments of the section. He also employs the assumption that the longitudinal flow is negligible so that the three dimensional problem can be reduced to a sequence of two dimensional ones. The result is a set of algebraic equat ons which can be solved for the source intensities, and hence, by employing (I -16), the velocity potentials. These results can then be used in (I l-15) to obta n the sea/structure interaction forces.

II.B FREQUENCY DOMAIN ANALYSIS

The results obtained in the previous section for the sea/structure interaction forces were in the frequency domain, while the formulation of the structural problem was in the time domain. While it is possible to transform the frequency domain forces to the time domain via an inverse Fourier transform, it is more efficient, if possible, to consider the structure response in the frequency domain. Thus, we first combine (I-5) with (II-15) to obtain

$$\bar{M} \ddot{\delta} = s + \eta q - [-\omega^2 H + i\omega D + R] \delta + \bar{d} \qquad (11\text{-}18)$$

then we take the Fourier transform of (II-18) to obtain

$$[-\omega^2 (\bar{M} + H) + i\omega D + R] \delta^* = \eta q^* \qquad (II\text{-}19)$$

where

$\bar{M}$ = structural mass matrix

H = hydrodynamic added mass matrix

D = hydrodynamic damping matrix

δ^* = complex motion

q^* = complex linear wave forces

η = the wave height

where a * is used to indicate a Fourier transform. Note that in (II-19) above, we have taken R to be evaluated about the mean vessel position, so that $s + \bar{d} = 0$, and δ^* is the dynamic response relative to a position of static equilibrium. If the vessel is not in equilibrium, then errors will result when computing the dynamic response.

Notice that q^*, the wave force, is complex and depends upon the wave direction, θ. Thus, if (II-19) is solved with $\eta = 1$, the result will be a complex vector, δ^*, which depends upon both the wave frequency and direction. This vector is called the response amplitude operator, or RAO, and it represents, of course, the response of the structure to a unit amplitude regular wave of frequency ω and heading 8. The actual response of the structure to this wave is obtained from the RAO as

$$\delta(t) = \text{Re}\ [\delta^*(\omega,\theta)\, e^{i\omega t}] \tag{II-20}$$

By replacing δ^* with its polar form, $|\delta^*| e^{i\omega\phi}$, where ϕ is a phase angle, we can write

$$\underset{\sim}{\delta}(t) = \text{Re}\ [\ |\underset{\sim}{\delta}^*| e^{i(\omega t + \phi)}] \tag{II-21}$$

Of course, since (II-19) is linear, the response of the structure to a sea composed of many regular waves can be obtained from superposition. In other words, if the sea can be represented by the sum of N waves $\eta_j(\omega_j,\theta_j)$, then.the structure response is *given* by

$$\underset{\sim}{\delta}(t) = \text{Re}\ [\sum_{j=1}^{N} \eta_j(\omega_j,\theta_j)\ \underset{\sim}{\delta}^*(\omega_j,\theta_j)] \tag{II-22}$$

<u>RAO's of Inertia Loads</u>

Once the vessel RAO's are obtained, the RAO's of the motions at other points and of the dynamic forces acting on bodies attached to the vessel can be easily obtained. To accomplish this, we will define

$$\underset{\sim}{\delta}^* = (\underset{\sim}{\overline{x}},\underset{\sim}{\theta}) \tag{II-23}$$

where $\bar{x}$ is the vector of RAO's of translation of the origin, and θ is the vector of RAO's of rotation. Again, assuming small angles, the RAO of the motion of a point r from the origin can be computed as

$$\underset{\sim}{x}_r = \underset{\sim}{\bar{x}} + \underset{\sim}{\theta} \times \underset{\sim}{r} \qquad \text{(II-24)}$$

where $\times$ denotes the vector cross product. To obtain the RAO's of the velocity and acceleration of the point, one simply multiplies (II-24) by $i\omega$ and $(i\omega)^2$ respectively. The RAO's of the motions of a point can be used to obtain the RAO's of the harmonic forces which act on a body whose center of gravity is located at the point by

$$\underset{\sim}{f} = M\,[\underset{\sim}{x}_r\omega^2 + g_c\underset{\sim}{\phi}]$$

$$\underset{\sim}{t} = \omega^2 \underset{\approx}{I}\underset{\sim}{\theta} \qquad \text{(II-25)}$$

where

$\underset{\sim}{f}, \underset{\sim}{t}$ = the RAO's of the force and torque on the body

M = the mass of the body

g_c = the acceleration of gravity

$\underset{\approx}{I}$ = the inertia matrix of the body

$\underset{\sim}{\phi}$ = a vector whose components are $(\theta_2, -\theta_1, 0)$

These forces and torques are represented in the vessel system, and this is the reason for the second term in the equation for the force. This term is a contribution from the weight of the body as the vessel pitches or rolls.

II.C SPECTRAL ANALYSIS

Since the response of the structure can be obtained from (II-22) once the sea is described, it would appear that the problem is solved. Unfortunately, we rarely have enough data to adequately describe the sea as required by (II-22). Instead, what is normally reported is the sea spectrum. The spectrum of the sea is a function which yields a measure of the energy in the sea as a function of frequency and direction. In other words, if the sea spectrum is given as $s(\theta,\omega)$, then

$$A = \int_{\theta_1}^{\theta_2}\int_{\omega_1}^{\omega_2} s(\theta,\omega)\, d\omega\, d\theta \qquad \text{(II-26)}$$

is a measure of the energy in the sea which has frequency between ω_1 and ω_2, and direction between θ_1 and θ_2.

For simplicity, suppose that the sea is uni-directional, i.e., that all waves come from a single direction. Then, mathematically, the sea spectrum is defined in terms of the Fourier transform of the autocorrelation of the wave amplitude as

$$s(\omega) = 2\pi \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \eta(t)\eta(t + \tau)\, dt \right] e^{-i\omega\tau}\, d\tau \qquad \text{(II-27)}$$

If this relationship is inverted,

$$\int_{-\infty}^{\infty} \eta(t)(t + \tau)\, dt = \int_{-\infty}^{\infty} s(\omega)\, e^{i\omega\tau}\, d\omega \qquad \text{(II-28)}$$

and for $\tau = 0$,

$$\sigma^2 = \int_{-\infty}^{\infty} \eta^2(t)\, dt = \int_{-\infty}^{\infty} s(w)\, d\omega \qquad \text{(II-29)}$$

so that the root mean square of the wave amplitude is simply the area under the spectrum.

The simple relationship (II-29) is quite important since it has been empirically established that the peaks in a sea follow a Rayleigh distribution. In other words the probability, P, of a peak exceeding η_0 is given by

$$P(\eta>\eta_0) = \int_{\eta_0}^{\infty} \frac{\xi}{\sigma^2} \exp(-\xi^2/2\sigma^2)\, d\xi, \quad \text{or}$$

$$P(\eta>\eta_0) = \exp\left(-\frac{\eta_0^2}{2\sigma^2}\right) \qquad \text{(II-30)}$$

Notice that this probability depends only on σ which can be obtained from the spectrum. Notice also that since our original problem is linear, the peaks in the structure response should follow the same distribution as the input.. Thus, if we could obtain the spectrum of the response, we could simply use (II-30) to obtain the probability that a given response will be exceeded.

Therefore, the goal is to obtain the spectrum of the structural response which is simply related to the sea spectrum by

$$s_0(\omega) = |x^*(\omega)|^2 \; s(\omega) \qquad \text{(II-31)}$$

Finally, for each degree of freedom of the structure, the RMS can be found by computing the area under the output spectrum, and the probability that any given value will be exceeded can be found from (II-30)

III. STRUCTURAL ANALYSIS

In Section II we outlined the method of solving the response of a rigid body subjected to a long-crested, unit amplitude wave train which yields the solution to (I-5). To determine the stresses in the structure, it is then necessary to solve equation (I-6) which yields the elastic deformation, $\underset{\sim}{u}$, of the structure which corresponds to the same wave train.

The primary difficulty in doing this is the inherent incompatibility between the hydrodynamic model and the structural model. This is most evident in the technical difficulties in deriving the structural wave loads, $\hat{g}$, in (I-6). The next two sections discuss the steps taken in OTTO to resolve the two models.

III.A STRUCTURAL MODELLING

OTTO employs standard finite element methods in modelling the elastic characteristics of the jacket and barge. The structure is idealized as a number of elastic elements which are interconnected at a finite number of nodes. Each node may have up to six degrees of freedom, those being three orthogonal translations and three orthogonal rotations, The structural system is then said to have N = 6 * NN total degrees of freedom, where NN is the total number of nodes.

The elements may be of three different types: beams, plates, or restraints. By assembling the stiffness characteristics of the elements, the system stiffness matrix is formed, where

$$\tilde{K}\,\tilde{u} = \tilde{F} \qquad \text{(III-1)}$$

and

K = the N x N stiffness matrix

u = vector of nodal displacements

F = applied forces .

III.6 COMPUTING THE STRUCTURAL WAVE LOADS

In Section II we discussed the calculation of the wave loads which act on a vessel. In order to perform a stress analysis, it is necessary to transfer these loads to the structure. In doing so, we must reconcile two distinct models -- the hydrodynamic model and the structural model. Sources of incompatibilities are

1.) The hydrodynamic model is based upon the shape of the wetted surface, whereas the structural model depends upon structural framing; and

2.) Structural models may be highly simplified, even to the point of a single beam at the vessel centerline.

Thus, each node on the hydrodynamic model does not, in general, have a corresponding node on the structural model to receive the load. Some type of mapping scheme must therefore be developed in order to transfer loads from the hydrodynamic model to the structural model.

In OTTO this is accomplished by taking the total load on a strip as the point of organization. The total load on a strip is found by integrating (II-10) over the wetted surface of the strip. This complex load is called F, where

$$\underset{\sim}{F}_S = \underset{\sim}{F}_{SR} + i\underset{\sim}{F}_{SI} \qquad \text{(III-2)}$$

where $\underset{\sim}{F}_{SR}$ and $\underset{\sim}{F}_{SI}$ are the real and imaginary parts of the load. These loads are to be mapped to a set of joints which are specified by the user when defining the shape of the strip. Since any number of nodes may be contained in this list, a least square technique is employed to obtain a distribution of the loads. If we limit consideration to the real load, $\underset{\sim}{F}_{SR}$, and partition this load into forces and moments as $\underset{\sim}{F}_{SR} = (\underset{\sim}{R},\underset{\sim}{M})$, then what we seek is a set of loads which minimize the function

$$C = \sum_{j=1}^{N} |\underset{\sim}{F}^j|^2 \qquad \text{(III-3)}$$

where N is the number of nodes which are to receive the load, and F^J is the force vector at the jth node. These nodal loads must satisfy the following equations

$$\sum_{j=1}^{N} F_i^j = R_i \qquad \text{(III-4)}$$

and

$$\sum_{j=1}^{N} \underset{\sim}{r}^j \times \underset{\sim}{F}^j = \underset{\sim}{M} \qquad \text{(III-5)}$$

where r^J is a vector from the vessel origin to the jth nodal point To solve this problem, we rewrite (III-5) as

$$C = \sum_{j=1}^{N} |F^j|^2_{\sim} + \underset{\sim}{\lambda}_1 \cdot [\sum_{j=1}^{N} F_i^j - R_i] + \underset{\sim}{\lambda}_2 \cdot [\sum_{j=1}^{N} \underset{\sim}{r}^j \times \underset{\sim}{F}^j - \underset{\sim}{M}] \qquad \text{(III-6)}$$

where $\underset{\sim}{\lambda}_1$, and $\underset{\sim}{\lambda}_2$ are LaGrange multipliers. Minimizing (III-6) yields a set of equations which is solved for the $\underset{\sim}{F}^J$. This procedure is repeated for the imaginary part, $\underset{\sim}{F}_{SI}$.

III.C RIGID BARGE IDEALIZATION

In OTTO, there is an option for treating the barge as a perfectly rigid body during the structural analysis in cases where the engineer feels that this idealization is warranted. In this case, the solution of (I-6) is simplified as follows: the structural wave loads, $\hat{g}$, are set to zero, and the structural displacements, u, involve only jacket node points. Thus,

$$\underset{\approx}{K}\,\underset{\sim}{u} = \underset{\sim}{\hat{d}} + \underset{\sim}{\hat{k}} - \underset{\approx}{M}\,\underset{\approx}{\bar{x}}\;\underset{\sim}{\ddot{\delta}} \qquad \text{(III-7)}$$

is solved for the jacket displacements, u. In constructing the right hand side of this equation, we will attack the problem on an element by element basis rather than formally constructing the rigid body transformation, $\underset{\approx}{\bar{x}}$. Note that $\ddot{\delta}$ can be decomposed as $\ddot{\delta} = (\underset{\sim}{\ddot{\chi}}, \underset{\sim}{\ddot{\phi}})$ where χ are the translations and ϕ are the Euler angles. The acceleration at some point, $\underset{\sim}{p} = (x, y, z)$, is then expressed from kinematics as

$$\underset{\sim}{\ddot{p}} = \underset{\sim}{\ddot{\chi}} + \underset{\sim}{\ddot{\phi}} \times \underset{\sim}{p} + \underset{\sim}{\dot{\phi}} \times (\underset{\sim}{\dot{\phi}} \times \underset{\sim}{p}) \quad . \qquad \text{(III-8)}$$

The inertial load on a member is computed by lumping one-half the member mass at each node, then multiplying by $-\ddot{p}$ above.

IV. SPECTRAL STRESS ANALYSIS

Having solved for the complex elastic deformation, u, of the system, it is then necessary to compute the stress RAO's. These stress RAO's are the basis for evaluating the fatigue life of the structure, and hence, it is necessary to incorporate empirical stress concentration factors (SCF's) when computing the stress RAO's. The following section details the development of the stress RAO's, and the computation of the cumulative damage ratios (CDR's).

IV.A CALCULATION OF MEMBER STRESS RAO'S

For a given wave case, i.e., period and direction, the member end complex reactions are computed. Next, for each of the N points around the circumference of a member, three complex normal stresses are determined: stress arising from axial load, stress arising from in-plane bending, and stress arising from out of plane bending. These stresses are called σ_A, σ_I, and σ_O, and may be written as

$$
\begin{aligned}
\sigma_A &= \sigma_A^R + i\sigma_A^I \\
\sigma_I &= \sigma^R + i\sigma^I \\
\sigma_O &= \sigma_O^R + i\sigma_O^I
\end{aligned}
\qquad \text{(IV-1)}
$$

The stress RAO at a given point is determined by factoring each of the above six stress components together with stress concentration factors (SCF's) based upon results of Kuang, et al [4], and Smedley [5].

The sequence of calculations to be performed at a joint are as follows:

1.) Determine all braces which lie in a plane. For each brace in this plane, compute the complex axial load, and in-plane and out-of-plane bending moments at the chord end of the brace. These are called P, M_I, and M_O,

2.) Based upon the real parts of P, M_I, and MD, determine the classification of the joint for that load path. The classification will be one of the following designations: 1.) K; 2.) T (or Y); or 3.) X (cross joint). Repeat this classification based upon the imaginary parts of P, M_I, and M_O.

3.) Determine the stress concentration factors, SCF, for both the real and imaginary load path classifications. The SCF's are calculated based upon the formulae given in Table IV-l, and the equations below, where SCF_A are used for stresses due to axial load only, and SCF, and SCFO are used for direct stress resulting from in-plane and out-of-plane moments only. Note that Kuang's results are used for K and T joints, and Smedley's are used for X joints.

$$SCF_A = \begin{cases} MAX\ (T3,\ T5) & ;\ \text{for K joint} \\ MAX\ (T1,\ T2) & ;\ \text{for T joint} \\ 1.33 * MAX\ (T1,\ T2) + 1; & \text{for X joint} \end{cases} \quad (IV\text{-}2)$$

$$SCF_I = \begin{cases} MAX\ (T13,\ T14) & ;\ \text{for K joint} \\ MAX\ (T11,\ T12) & ;\ \text{for T joint} \\ 1.33 * AMAX\ (T11,\ T12) & ;\ \text{for X joint} \end{cases} \quad (IV\text{-}3)$$

$$SCF_O = \begin{cases} MAX\ (T15,\ T17);\ \text{for K joint where } d/D<0.55 \\ MAX\ (T16,\ T18);\ \text{for K joint where } d/D>0.55 \\ \text{Same as K joint; for T joint} \\ 1.33 * \text{above value; for X joint} \end{cases} \quad (IV\text{-}4)$$

4.) For each stress point around the circumference of the brace, calculate the complex axial stress σ_A, and the complex direct stresses resulting from in-plane and out-of-plane moments, σ_I and σ_O.

5.) The total stress RAO for each point is then calculated from

$$RAO^2(\sigma) = [(\sigma_A^R SCF_A^R + \sigma SCF_I^R + \sigma_O^R SCF_O^R)^2 + (\sigma_A^I SCF_A^I + \sigma_I^I SCF_I^I + \sigma_O^I SCF_O^I)^2] \quad (IV-5)$$

where the superscripts R and I refer to the real and imaginary parts, respectively.

TABLE IV-1

FORMULAE FOR ESTIMATING SCF IN TUBULAR JOINTS

$$T1 = 1.177(T/D)^{-.808} e^{-1.2(d/D)^3} (t/T)^{1.333} (D/L)^{-.057} \sin^{1.694}\theta$$

$$T2 = 2.784(T/D)^{-.55} e^{-1.35(d/D)^3} (t/T)(D/L)^{-.12} \sin^{1.94}\theta$$

$$T3 = .949(T/D)^{-.666} (d/D)^{-.059} (t/T)^{1.104} (g/D)^{.067} \sin^{1.521}\theta$$

$$T5 = .825(T/D)^{-.157} (d/D)^{-.441} (t/T)^{.56} (g/D)^{.058} e^{1.448 \sin\theta}$$

$$T11 = .463(T/D)^{-.6} (d/D)^{-.04} (t/T)^{.86} \sin^{.57}\theta$$

$$T12 = 1.109(T/D)^{-.23} (d/D)^{-.38} (t/T)^{.38} \sin^{.21}\theta$$

$$T13 = 1.4(T/D)^{-.38} (d/D)^{.06} (t/T)^{.94} \sin^{.9}\theta$$

$$T14 = 2.827(T/D)^{-.35} (t/T)^{.35} \sin^{.5}\theta$$

$$T15 = .465(T/D)^{-1.014} (d/D)^{.787} (t/T)^{.889} \sin^{1.557}\theta$$

$$T16 = .199(T/D)^{-1.014} (d/D)^{-.619} (t/T)^{.889} \sin^{1.557}\theta$$

$$T17 = .803(T/D)^{-.852} (d/D)^{.801} (t/T)^{.543} \sin^{2.033}\theta$$

$$T18 = .42(T/D)^{-.852} (d/D)^{-.281} (t/T)^{.543} \sin^{2.033}\theta$$

where

D,T = diameter and thickness of chord

d,t = diameter and thickness of brace

L = length of joint can (assumed 10 ft.)

g = gap distance (assumed 2 in.)

IV.B CALCULATION OF CUMULATIVE DAMAGE RATIOS (CDR'S)

The damage incurred in a given seastate, s, is denoted CDR_s. Thus, it follows that, for a transit consisting of NS different seastates, the total cumulative damage ratio, CDR, is

$$CDR = \sum_{s=1}^{NS} CDR_s \qquad \text{(IV-6)}$$

CDR_s is computed as follows: First, note that the stress RAO at a given point in the structure is denoted SRAO (ω,θ). Thus, CDR_s for this stress is

$$CDR_s = \frac{T}{\tau_{AV}} \int_0^\infty \frac{P(r)}{N(r)} \, dr \qquad \text{(IV-7)}$$

where

T = Time duration of the seastate

τ_{AV} = average period for stress variation

$P(r)$ = probability density function of the stress range

$N(r)$ = average number of cycles to failure at a given stress range, r.

The above formula is, of course, a continuous form of Miner's rule for cumulative damage. The terms are computed from the following equations:

$$\tau_{AV} = 2\pi \left[\frac{m_0}{m_2} (1 - \varepsilon^2)\right]^{\frac{1}{2}} \qquad \text{(IV-8)}$$

$$\varepsilon^2 = (m_0 m_4 - m_2^2)/(m_0 m_4) \qquad \text{(IV-9)}$$

$$P(r) = \frac{r}{4m_0} e^{-r^2/8m_0} \qquad \text{(IV-10)}$$

$$N(r) = Ar^{-J} \qquad \text{(IV-11)}$$

and the spectral moments, m_i, are defined as

$$m_i = \int_{\theta=\bar{\theta}-\pi/2}^{\bar{\theta}+\pi/2} \int_{\omega=0}^{\infty} \omega^i \, |SRAO(\omega,\theta)|^2 \, S_{HH}(\omega) \cos^2(\theta-\bar{\theta}) \, d\omega d\theta \qquad \text{(IV-12)}$$

where

S_{HH} = the wave spectrum

$\overline{\theta}$ = mean wave heading

z = spreading function

ω = wave frequency

θ = wave heading

IV.C JOINT CLASSIFICATION

Note that the stress RAO depends upon the SCF, which is in turn dependent upon the joint classification, i.e., K, T, or X. That the joint classification is dependent upon the geometry of the joint and also the load path is pointed out in API-RP 2A [6], but the method of classing the joint is not spelled out unambiguously. This section is intended to define this classification in a form suitable for implementation.

We begin by making the following definitions:

$$S = (SCF_K, SCF_T, SCF_X) \tag{IV-13}$$

$$\alpha = (a_K, \alpha_T, \alpha_X) \tag{IV-14}$$

where SCF_K, SCF_T, and SCF_X are the stress concentration factors, defined in Section IV.A, and a_K, α_T, and a_X are components of the joint classification factors. Thus, the SCF to be applied to a given brace is

$$SCF = \underset{\sim}{\alpha} \cdot \underset{\sim}{S} \tag{IV-15}$$

Note that the idea here is that a joint may transfer loads in more than one mode depending upon the load path.

Next, we make some additional definitions:

l_i = the transverse resultant load on the ith member on the "left" side of the chord

r_i = the transverse resultant load on the ith member on the "right" side of the chord

where we consider only those members that, taken with the chord, lie in the same plane. Further, we define

L^+ = the sum of all l_i which are positive

L^- = the sum of all l_i which are negative

R^+ = the sum of all r_i which are positive

R^- = the sum of all r_i which are negative

and

$$L = L^+ + L^-$$

$$R = R^+ + R^- \qquad \text{(IV-16)}$$

$$V = R + L.$$

The above parameters form the basis for classing a jo nt. The decisions to be made will be based upon the following criteria:

1.) The examples shown in [6] will yield the same resulting classifications.

2.) A member will be classed as K if possible.

3.) Loads which are transferred from one side of the chord to the other side will be evenly distributed among ail members involved.

4.) The classification will be well-behaved, i.e., small changes in load path will produce small changes in α.

The majority of decisions are based upon L, R, and V. If we restrict our attention to the braces on the left, then, if L = 0, no shear is transferred through or to the chord from this side, and therefore, all members on the left are classed K, so that α = (1 ,O,O). If $L \neq 0$, then the situation is more complex, and some of the members will be classed as T or X depending upon the total shear, V. The classifications made in OTTO are summarized in Table IV-2 for cases where $L > 0$. Of course, all other cases can be constructed from the basic cases in this table.

TABLE IV-2

JOINT CLASSIFICATION FACTORS, a = $(\alpha_K, \alpha_T, \alpha_X)$

	Case 1 L=0	Case 2 L > 0; V = 0	Case 3 L > 0; V > 0	Case 4 L > 0; V < 0
$l_i < 0$*	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)
$l_i > 0$	(1,0,0)	$[1-\alpha_X, 0, (L/L^+)]$	$[1-\alpha_T, (L/L^+), 0]$	$[1-(\alpha_T+\alpha_X), \|(L+V)/L^+\|, \|V/L^+\|]$

* $l_i < 0$ means all braces with transverse load are less than zero.

V. MAXIMUM STRESS ANALYS IS

In addition to fatigue failure criteria, it is sometimes important to consider the stress in the structure due to a single event, such as a severe seastate of short duration. In this case, a strength criteria may take precedence over a fatigue criteria.

In OTTO, this problem is addressed by allowing the treatment of the structure statically. By this, we mean that a snapshot is taken of the barge/jacket combination in the time-domain, and a static analysis is performed. Advantages of this approach are

1.) The non-linear interface between the jacket and barge can be treated properly (i.e., gap elements).

2.) Static forces, such as wind, current, etc., can be treated.

V.A TIME DOMAIN LOADS

For time domain results in irregular seas, the wave loads and rigid body accelerations must be constructed. Since (II-19) is linear, the response of the structure to a sea of many regular waves can be obtained by superposition. Thus, if the sea can be represented by a sum of N regular waves, $\eta_j(\omega_j,\theta_j)$, then the response of any given quantity, Q, may be expressed in terms of its RAO's, Q^*, by

$$Q(t) = \text{Re}\left[\sum_{j=1}^{N} \eta_j(\omega_j,\theta_j)Q^*(\omega_j,\theta_j)\right] \quad \text{(V-1)}$$

If the irregular wave height is expressed similarly as

$$\eta(t) = \text{Re}\left[\sum_{j=1}^{N} |\eta_j| \, e^{i(\omega_j t + \phi_j)}\right] \quad \text{(V-2)}$$

then the amplitudes are found from the wave spectrum, S, by

$$|\eta_j| = 2\int_{\theta_1}^{\theta_2}\int_{\omega_1}^{\omega_2} S(\omega,\theta)\, d\omega d\theta \quad \text{(V-3)}$$

where the phase angles, ϕ_j, are chosen arbitrarily, and the limits of integration are

$$\begin{aligned}
\omega_1 &= (\omega_j - \omega_{j-1})/2 \\
\omega_2 &= (\omega_{j+1} - \omega_j)/2 \\
\theta_1 &= (\theta_j - \theta_{j-1})/2 \\
\theta_2 &= (\theta_{j+1} - \theta_j)/2
\end{aligned} \quad \text{(V-4)}$$

V.B GAP ELEMENT

During the transit of a jacket, the jacket is only partially connected to the barge, i.e., it is welded to the barge at only a few tie-down locations. At other locations along the launchway, the jacket is free to slide along the launchways or even lift off the launchway completely. OTTO has the ability to analyze this problem by using a specialized element that is called a gap element. In this section, we will outline the general features of this element, and the method of solution.

Statement of the Gap Problem

The problem in general is that of two or more structures wh ch are hooked together in some fashion. Here, "in some fashion" is taken to mean gap elements. A gap element is deal ized as two nodes in space that are constrained together by the ability of the element to transmit internal loads.

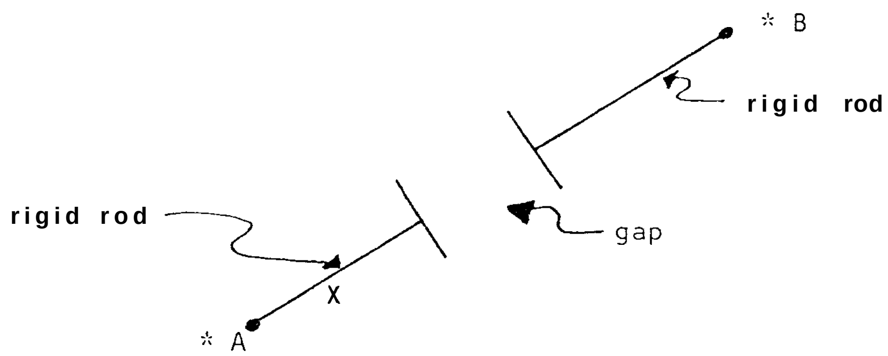

Figure V-I
Gap Element Coordinates

The X-axis of the member is taken as its axial degree of freedom; the Y and Z-axes are perpendicular to this axis. The features of the element are now described.

Let the vectors $\underset{\sim}{u}_A$, and $\underset{\sim}{u}_B$, contain the deflection, in the element local system (X,Y,Z), of the two nodes * A and * B. Thus, the axial "stretch" of the element becomes

$$u_{1B} - u_{1A} \qquad \text{(V-5)}$$

Since the element is rigid when the gap is closed, we can write the following constraint equation

$$u_{1B} - u_{1A} \geq 0 \qquad \text{(V-6)}$$

and the element must generate internal loads to satisfy (V-Z). Thus, it is seen that this element is non-linear in that its "stiffness" depends upon whether or not the gap is closed.

<u>Equations of Constraint</u>

The equilibrium equation involving nodes * A and * B is

$$\underset{\approx}{K}\,\underset{\sim}{x} = \underset{\sim}{F} \qquad \text{(V-7)}$$

Where

x = deflections of the two nodes, in local system

F = forces at the nodes

K = stiffness matrix

Now, if the gap element is closed, no al 1 of the x_i are independent degrees of freedom since equation (V-6) must be satisfied. If we partition the deflection vector x so that

$$x = (x_i, x_d) \qquad \text{(V-8)}$$

where x_i are the independent degrees of freedom, and x_d are the dependent degrees of freedom, then equation (V-7) may be written as

$$\begin{bmatrix} K_{ii} & K_{id} \\ K_{di} & K_{dd} \end{bmatrix} \begin{Bmatrix} x_i \\ x_d \end{Bmatrix} = \begin{Bmatrix} F_i \\ F_d \end{Bmatrix}$$

The constraints among the x_i may be wr ten as

$$x_d = G\, x_i \qquad \text{(V-9)}$$

which can be used to augment the equilibrium equation so tha

$$\begin{bmatrix} K_{ii} & K_{id} & G^T \\ K_{di} & K_{dd} & -I \\ G & -I & 0 \end{bmatrix} \begin{Bmatrix} x_i \\ x_d \\ q_d \end{Bmatrix} = \begin{Bmatrix} F_i \\ F_d \\ 0 \end{Bmatrix} \qquad \text{(V-10)}$$

where q_d is a set of forces required to satisfy equations (V-7) and (V-9). Solving (V-10) yields

$$K^* x_i = F^* \qquad \text{(V-11)}$$

$$q_d = K_{di}\, x_i + K_{dd}\, x_d - F_d \qquad \text{(V-12)}$$

where

$$K^* = T^T\, K\, T \qquad \text{(V-13)}$$

$$F^* = F_i + T^T\, F_d \qquad \text{(V-14)}$$

and

$$T = \begin{Bmatrix} I \\ G \end{Bmatrix} \qquad \text{(V-15)}$$

where I is the identity matrix whose order is the same as that of x_i.

Solution Algorithm

The algorithm for solving the gap problem is outlined in the following steps.

1.) Form the $\underset{\approx}{K}$ and F matrices as in Equation (V-7).

2.) For each gap element, g (g = 1...., number of gaps), initialize the constraint matrix, t^g, and the internal loads, as follows:

$$\underset{\approx}{t}^g = \begin{bmatrix} I_6 \\ \hline I_6 \end{bmatrix} \quad ; \quad \underset{\sim}{f}^g = 0$$

where $\underset{\approx}{I}_6$ is the 6 x 6 identity matrix. This effectively forces each gap to a "closed" position.

3.) The system constraint matrix is calculated as

$$\underset{\approx}{T} = \sum_{gaps} \underset{\approx}{\beta}^g \underset{\approx}{t}^g$$

where $\underset{\approx}{\beta}$ transforms a vector from the local (x,y,z) system to the global coordinate system.

4.) Solve the constrained system of equations for the displacements, $\underset{\approx}{x}$, and constraint forces, $\underset{\sim}{q}_d$,

$$\underset{\approx}{T}^T \underset{\approx}{K} \underset{\approx}{T} \underset{\sim}{x}_i = \underset{\approx}{T}^T \underset{\sim}{F}$$

$$x = \underset{\approx}{T} \underset{\sim}{x}_i$$

$$\underset{\sim}{q}_d = \underset{\approx}{K}_{di} \underset{\sim}{x}_i + \underset{\approx}{K}_{dd} \underset{\sim}{x}_d - \underset{\sim}{F}_d$$

5.) If the solution is admissable, that is, if all gap elements are in compression or have zero loads, and all displacement bounds are met, then a solution has been found. If not, then generate a new set of element constraint matrices, $\underset{\approx}{t}^g$, and return to Step 3.

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