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Assessing the Integrity of a Body Subjected to a Seaway

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I. INTRODUCTION

One of the most interesting aspects of Offshore Engineering is assessing the structural integrity of a body subjected to a seaway. While there are two ways of viewing the problem, in the frequency domain and in the time domain, most people adopt one method to the exclusion of the other. This is unfortunate since each has situations in which it is clearly superior to the other.

Here, we present a brief overview of the problem with an emphasis on the frequency domain, which is the method most often neglected.

II. THE MODEL OF THE SEA

The behavior of the sea is stochastic. By this, we mean that one rarely is given enough information to completely define its behavior. Instead, what is normally reported is the sea spectrum. The spectrum of the sea is a function which yields a measure of the energy in the sea as a function of frequency and direction. In other words, if the sea spectrum is given as $S(\theta, \omega)$, then

$$A = \int_{\theta_1}^{\theta_2} \int_{\omega_1}^{\omega_2} S(\theta, \omega)\, d\omega\, d\theta \tag{II.1}$$

is a measure of the energy in the sea which has frequency between ω_1 and ω_2, and direction between θ_1 and θ_2. Mathematically, the sea spectrum is defined in terms of the auto-correlation of the wave amplitude as

$$S(\omega) = 2\pi \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} \eta(t)\, \eta(t+\tau)\, dt \right] e^{(-i\omega\tau)}\, d\tau \tag{II.2}$$

for a unidirectional sea. Likewise, one can express the temporal nature of the sea with given spectrum as

$$\eta(t) = Re \left[\sum_{j=1}^{N} \eta_j\, e^{(i\omega_j t + \phi_j)} \right] \tag{II.3}$$

where

$$|\eta_j|^2 = 2 \int_{\omega_1}^{\omega_2} S(\omega)\, d\omega \tag{II.4}$$

and there are not restrictions on the parameters ϕ_j. In other words, there are many seas which will yield the same spectrum.

It is the lack of uniqueness of a sea corresponding to a given spectrum which creates difficulties. If one is to assess the integrity of a body subjected to a given storm, he has many sea samples from which to choose. There are basically two alternatives: check several samples, or consider the problem stochastically. Here, by stochastically, we mean that instead of seeking the solution, we seek a solution which has a specified probability of being exceeded. For the sea, it has been empirically established that the peaks reasonably follow a Raleigh distribution. In other words, the probability, P, of a peak exceeding η_0 is given by

$$P(\eta > \eta_0) = \exp\left(-\frac{\eta_0^2}{\sigma^2} \right) \tag{II.5}$$

where

$$\sigma^2 = \int_{-\infty}^{+\infty} \eta^2(t)\, dt = \int_{-\infty}^{+\infty} S(\omega)\, d\omega\ , \tag{II.6}$$

Thus, if we know the spectrum, we can find any probability of exceedence.

While either of the above two approaches yield viable results, our primary purpose here will be to investigate the probabilistic approach.

III. THE DYNAMICS PROBLEM

For our purposes, we will suppose that:

1. The motions of the body are small,
2. The fluid flow in the sea is inviscid and irrotational,
3. The acceleration of the deformation is negligible, and
4. The sea/structure interaction forces are independent of the deformation.

Under these premises, the equations of motion of the body can be written as

$$\frac{d}{dt}\Big[\mathbf{I}\,\dot{\mathbf{y}}\Big] + \mathbf{K}\,\mathbf{y} = \hat{\mathbf{g}} + \hat{\mathbf{d}} + \hat{\mathbf{k}} \tag{III.1}$$

where $\mathbf{y}$ defines the configuration of the body, $\mathbf{I}$ is the inertia matrix, $\mathbf{K}$ is the stiffness matrix, $\hat{\mathbf{g}}$ is the force on the body due to the sea, $\hat{\mathbf{d}}$ is the weight, and $\hat{\mathbf{k}}$ is any other force on the body.

Now, let us define a decomposition

$$\mathbf{y} = \mathbf{V}\mathbf{x} + \mathbf{u} \tag{III.2}$$

where $\mathbf{x}$ is a rigid motion of the body, $\mathbf{u}$ is the deformation of the structure, and $\mathbf{V}$ is a transformation which transforms the rigid motion of the reference point into a rigid motion at each degree of freedom. Using this decomposition results in

$$\frac{d}{dt}\Big[\mathbf{I}\mathbf{V}\dot{\mathbf{x}} + \mathbf{I}\dot{\mathbf{u}}\Big] + \mathbf{K}\mathbf{V}\mathbf{x} + \mathbf{K}\mathbf{u} = \mathbf{f}\ , \tag{III.3}$$

where $\mathbf{f}$ is simply the net force on the body. If we now employ the assumptions concerning small motions and negligible deformation acceleration, (III-3) becomes

$$\mathbf{I}\mathbf{V}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{V}\mathbf{x} + \mathbf{K}\mathbf{u} = \mathbf{f}\ . \tag{III.4}$$

We will now limit our attention to systems where the stiffness matrix, K, contains no connections to ground. Mathematically, this restriction can be expressed as

$$\dot{\mathbf{z}} \cdot \mathbf{K}\mathbf{u} = 0 \tag{III.5}$$

for any deformation $\mathbf{u}$ and any rigid motion $\mathbf{z}$. The equations of motion can now be decomposed into two systems

$$\mathbf{V}^T\mathbf{I}\mathbf{V}\ddot{\mathbf{x}} = \mathbf{V}^T\mathbf{f}\ , \mathbf{K}\mathbf{u} = \mathbf{f} - \mathbf{I}\mathbf{V}\ddot{\mathbf{x}}\ . \tag{III.6}$$

This result shows that, within the limitations imposed, the original problem can be viewed as a rigid body dynamics problem and a structural analysis problem with the addition of the rigid body inertia loads.

For convenience, we will rewrite the first of (III.6) as

$$\bar{\mathbf{I}}\ddot{\mathbf{x}} = \bar{\mathbf{f}} = \bar{\mathbf{g}} + \bar{\mathbf{d}} + \bar{\mathbf{k}} \tag{III.7}$$

where the quantities with a superposed - are the total quantities for the body.

IV. THE HYDRODYNAMICS PROBLEM

A basic ingredient in the equations of motion is the sea/structure interaction force. To evaluate this force, one integrates the pressure over the submerged surface of the body. Here, the pressure is given by the linearized Bernoulli equation

$$p = -\rho\Big(\mathbf{p}\phi + gz\Big) \tag{IV.1}$$

where ρ is the fluid density, g is the acceleration of gravity, z is the depth of submergence, and ϕ is the velocity potential for the flow. The conditions on ϕ are

$$\begin{aligned} \bigtriangledown^2 \phi &= 0 && \text{on the exterior of the body,} \\ \frac{\partial^2\phi}{\partial t^2} + g\frac{\partial\phi}{\partial z} &= 0 && \text{on the free surface,} \\ \nabla\phi\cdot\mathbf{n} &= \dot{\mathbf{y}}\cdot\mathbf{n} && \text{on the body surface,} \end{aligned} \tag{IV.2}$$

and an appropriate initial condition. Notice that the body boundary condition is expressed in terms of the velocity and hence, the hydrodynamic problem is formally coupled to the dynamics problem.

At this point, it is convenient to decompose the velocity potential as

$$\phi = \phi_i + \phi_d + \mathbf{\Phi}\cdot\dot{\mathbf{x}} \tag{IV.3}$$

where all ϕ's satisfy LaPlace's Equation and the free surface condition, ϕ satisfies a radiation condition, and, on the body boundary

$$\nabla\phi_i\cdot\mathbf{n} = \nabla\phi_d\cdot\mathbf{n}\ ,$$

and

$$\nabla\mathbf{\Phi}\cdot\mathbf{n} = \mathbf{T}\,\mathbf{n} \tag{IV.4}$$

Here, $\mathbf{T}$ is a matrix which transforms the velocity at the reference point into velocities at each point on the body surface. Notice that by this decomposition, we have decoupled the hydrodynamics problem from the dynamics problem. Also, ϕ_i is the incident wave potential which is assumed to be known, ϕ_d is the diffraction potential, and ϕ is the radiation potential.

Now, the force on a portion of the body, P, can be expressed as

$$g = -\rho\int_{\partial P}\Big(\mathbf{p}\phi + gz\Big)\,ds\ ,$$

or

$$\tag{IV.5}$$

$$\mathbf{g} = \mathbf{H}\ddot{\mathbf{x}} - \mathbf{D}\dot{\mathbf{x}} - \rho \int_{\partial P} \Big(gz + \mathbf{p}\phi_i + \mathbf{p}\phi_d \Big) \mathbf{T}\mathbf{n}\, ds$$

where

$$\mathbf{H} = \rho \int_{\partial P} \left(\mathbf{\Phi} \otimes \frac{\partial \mathbf{\Phi}}{\partial \mathbf{n}} \right) ds$$

$$\mathbf{D} = \rho \int_{\partial P} \left(\mathbf{p\Phi} \otimes \frac{\partial \mathbf{\Phi}}{\partial \mathbf{n}} \right) ds \tag{IV.6}$$

Of course, $\mathbf{H}$ and $\mathbf{D}$ are called the added mass and damping matrices of the part of the body.

While we have obtained a representation of the force, the problem remains to actually solve for the potentials. While this solution is not our concern, it should be mentioned that this can be accomplished in several ways. For many bodies, it has been found that an approximation of neglecting the free surface condition and approximating the radiation potentials yields satisfactory results. This particular method is called Morison's Equation. Alternatively, several techniques have been developed for solving the problem more precisely. These techniques are called diffraction theories.

Before leaving the question of hydrodynamics, let us rewrite the expression for the force on a part as

$$\mathbf{g} = -\mathbf{H}\ddot{\mathbf{x}} - \mathbf{D}\dot{\mathbf{x}} + \mathbf{i} + \mathbf{b} \tag{IV.7}$$

where

$$\mathbf{i} = -\rho \int_{\partial P} \Big(\mathbf{p}\phi_i + \mathbf{p}\phi_d \Big)\, \mathbf{T}\,\mathbf{n}\, ds$$

is the wave exciting force, and

$$\mathbf{b} = -\rho g \int_{\partial P} z\, ds \tag{IV.8}$$

is the buoyancy.

V. FREQUENCY DOMAIN ANALYSIS

Traditionally, the motion of bodies subjected to a seaway has been studied in the frequency domain. In other words, one supposes that there is a mean position, $\bar{\mathbf{x}}$, and one looks for a solution $\mathbf{x}^*$ which is the motion of the body about the mean in a regular sea of unit amplitude and given heading and period. In other words,

$$-\omega^2\bar{\mathbf{I}}\mathbf{x}^* = \bar{\mathbf{f}}(\bar{\mathbf{x}})\frac{\partial\bar{\mathbf{f}}(\bar{\mathbf{x}})}{\partial\mathbf{x}}\mathbf{x}^* \tag{V.1}$$

where we have symbolically shown the dependence of the force on the configuration. If we use the various representations in the above, there results

$$[-\omega^2(\bar{\mathbf{I}}+\mathbf{H}) + i\omega\mathbf{D} + \mathbf{R}]\mathbf{x}^* = \mathbf{i}^* \tag{V.2}$$

where $\mathbf{i}^*$ is the wave exciting force for the period and heading, and

$$\mathbf{R} = \frac{\partial\mathbf{f}(\bar{\mathbf{x}})}{\partial\mathbf{x}} \tag{V.3}$$

Notice that the solution to (V-2) is a complex vector. Using this decomposition in the structural equations yields

$$\begin{aligned} \mathbf{K}\bar{\mathbf{u}} &= \mathbf{f}(\bar{\mathbf{x}}) \\ \mathbf{K}\bar{\mathbf{u}}^* &= \mathbf{f}(\bar{\mathbf{x}}^*) - \bar{\mathbf{I}}\mathbf{V}\mathbf{x}^* \end{aligned} \tag{V.4}$$

so that we can have one structural load case for the mean and two for each wave heading and period.

Notice that since our equations are linear, superposition holds. Thus, suppose that we have a mean solution $\mathbf{u_o}$ and several solutions for headings and frequencies. Then, if the wave amplitude is given by

$$\eta(t) = \sum\sum \eta_{ij}\, e^{(i\omega_i t + k\cos\theta_j)}$$

the structural response will be

$$\mathbf{u(t)} = \mathbf{u}_0 + \sum\sum \mathbf{u_{ij}}\,\eta_{ij}\, e^{(i\omega_i t + k\cos\theta_j)} \tag{V.5}$$

Alternatively, we can define the deformation spectrum as we did the sea spectrum, and we find that

$$S_u = |\mathbf{u}^*|^2 S_\eta$$

In other words, the deformation spectrum is related to the sea spectrum by the deformation response operators. In a like manner, we can define the spectrum of any quantity <u>*linearly*</u> related to the deformations. Once we have the spectrum, we can then assess the probability of given values being exceeded as was discussed in Section II.

VI. FATIGUE

The assessment of fatigue is normally expressed by a cumulative damage ratio. In other words, by Miner's Rule

$$CDR = \frac{T}{t} \int_0^\infty \frac{P(r)}{N(r)} dr \tag{VI.1}$$

where CDR is the cumulative damage ratio, T is the duration of a process, t is the average period for a stress cycle, P is the probability density function of the stress range, and N is the average number of cycles to failure at a given stress range. Notice that if a body is subjected to several different sea states, then the total damage ratio can be obtained by adding the CDR's for each sea state.

Notice that the frequency domain is an ideal place to consider the fatigue problem. As discussed previously, once the deformation response operators have been computed, the stress spectrum is simply

$$S_s = |S^*|^2 S_\eta \tag{VI.2}$$

where S^* is the stress response operator and S_η is the sea spectrum. Now, using the Raleigh distribution

$$\begin{aligned}
P(r) &= \frac{r}{4m_0} \exp\left(\frac{-r^2}{8m_o}\right), \\
t &= 2\pi[\frac{m_0}{m_2}(1-\epsilon^2)]^{\frac{1}{2}}, \\
\epsilon^2 &= (m_0 m_4 - m_2^2)/m_0 m_4 \text{ , and} \\
m_j &= \int_0^{2\pi} \int_0^\infty S_s(\omega, \theta)\, \omega^j \, d\omega \, d\theta
\end{aligned} \tag{VI.3}$$

Thus, the cumulative damage is easily computed from the stress response operators.

VII. MEMBER FAILURE

To assess the integrity of a given member, one normally employs a criteria which is a nonlinear function of the various element stresses. Because of this nonlinearity, one cannot form a "spectrum of the criteria" and some approach other than using the Raleigh Distribution on the spectrum must be employed. One obvious approach would be to create a time domain sample. Here, one has a deterministic set of stresses and he can proceed as normal. The difficulty with this approach is deciding upon the number of deterministic cases to consider. If too many are chosen, the cost can become prohibitive; if too few, there is serious doubt as to whether a situation close to the critical has been investigated.

An alternative to the time domain synthesis is to construct a set of element stresses as

$$s = s_m + sign(s_m)|s_p| \tag{VII.1}$$

where s_m is the mean stress and s_p is the stress which has a given probability of being exceeded. While this will definitely yield something which can be used in a deterministic failure criteria, it is not obvious that it will give answers which are not overly conservative.

To compare the two approaches, the small jacket shown in Figure 1 was analyzed using both methods. First, a stress analysis was performed in the frequency domain with four headings and seventeen periods (137 load cases). A time domain sample of the total inertia load on the structure was synthesized for twelve sea states, and the time when any force had a maximum or minimum were chosen to perform both member and joint checks. This resulted in 114 cases to be considered. Alternatively, the same twelve sea states were used to generate stresses according to (VII-1). It was found that the stochastic code check ran in 20% of the computer time required for the time domain samples. The corresponding ratio for joint checks was 37%.

When performing these code checks stochastically, the exceedence criteria employed was 1/100. Thus, one would expect that there would be some cases where the time domain results would be greater. This was found to be the case for 11% of the members and 14% of the joints. A more important comparison, however, is for those members and joints where the code check ratio was greater than one. For joints, one out of 24 was governed by the time domain; for members, one out of eighteen. Perhaps the most important comparison, however, is between the magnitudes of the two sets of results. For joints, the maximum difference between the two ratios was 1.57 for the stochastic vs. 1.48 for the time domain. For members, it was 8.550 for stochastic vs. 8.017 for the time domain.